Machine learning – acceleration of global optimization

Bjørk Hammer

IKZ & FAIRMAT winter school – Jan, 2023



Outline

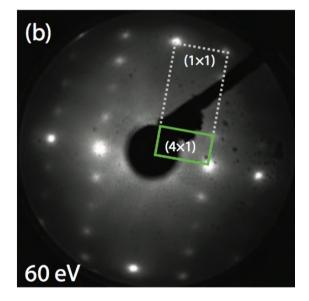
Motivation

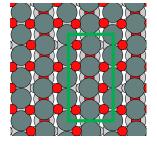
Machine learning learning of interatomic potentials

Learning chemistry? Artificial intelligence...

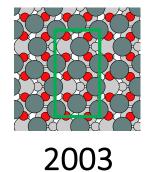
Why global optimization is needed

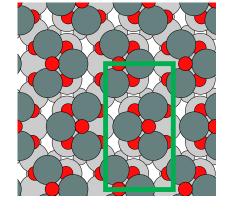
Determine structure Reduced tin-oxide, SnO₂, surface





2001

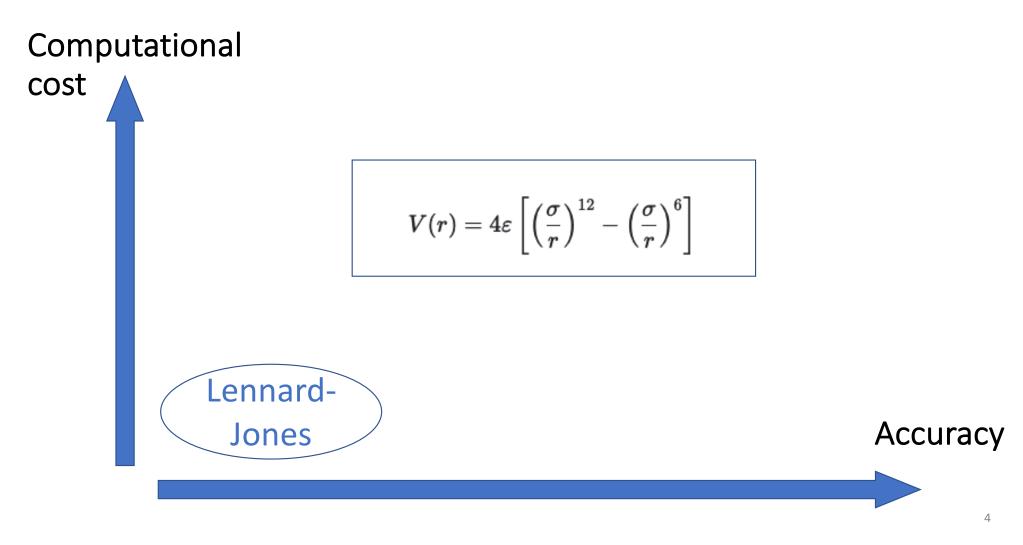




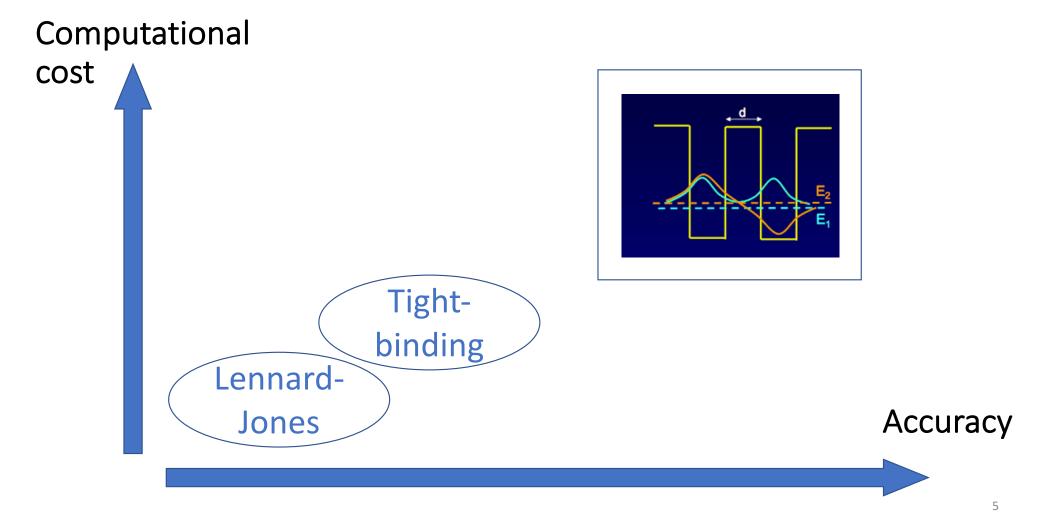
2017

3

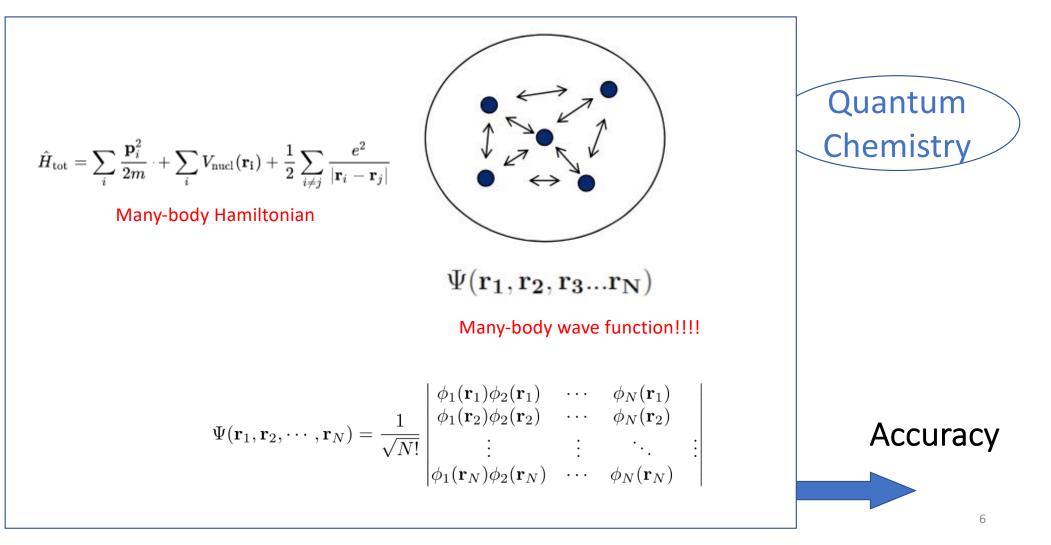
Total energy methods



Total energy methods

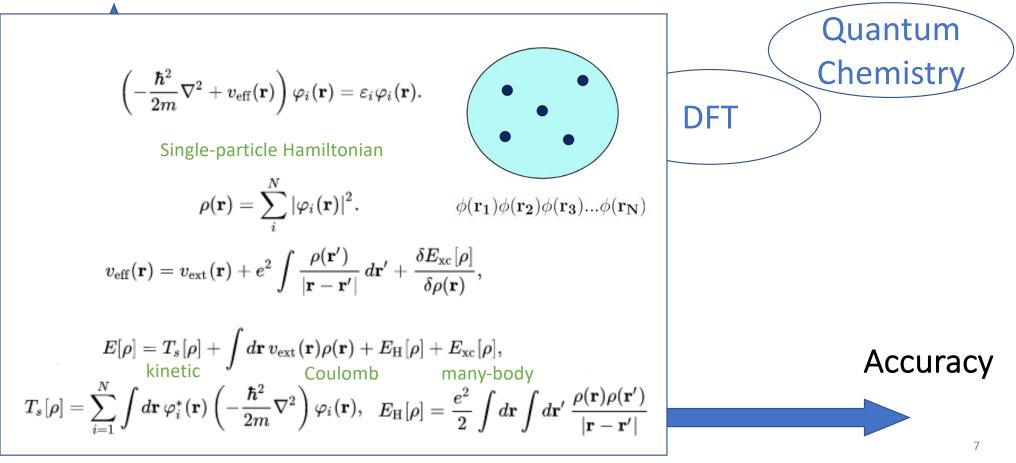


Total energy methods

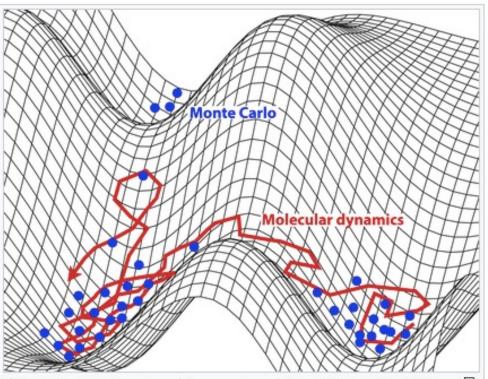


Density functional theory

Computational cost



Simulation methods



Schematic representation of the sampling of the system's potential 5 energy surface with molecular dynamics (in red) compared to Monte Carlo methods (in blue).

Figure credit: Wikipedia

Molecular dynamics:

$$egin{aligned} F(X) &= -
abla U(X) = M \dot{V}(t) \ V(t) &= \dot{X}(t). \end{aligned}$$

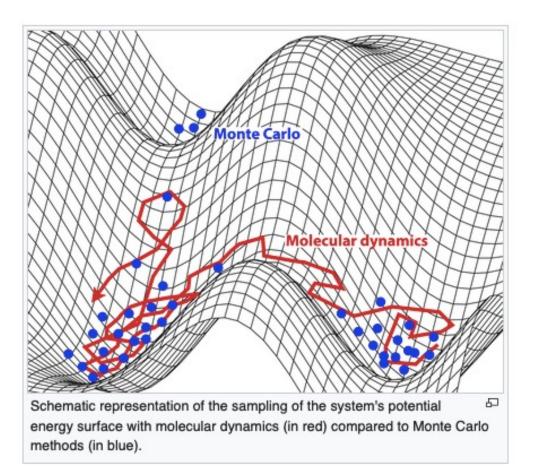
Metropolis Monte-Carlo:

- Perturb the structure ٠
- Accept if potential energy decreases, $\Delta U < 0$

or

$$n < \exp(-\Delta U/\Delta U_0)$$
 .

Simulation methods



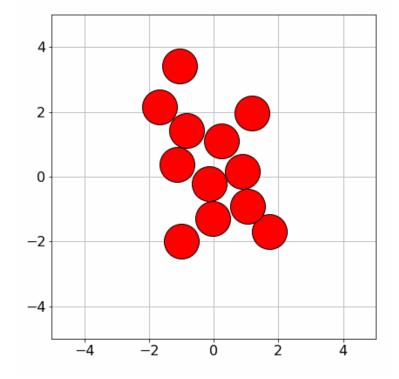


Figure credit: Python course at Aarhus University

Figure credit: Wikipedia

Computational times for DFT simulations on a supercomputer

100 atoms on 8 CPU cores

Energy evaluation	1 hrs
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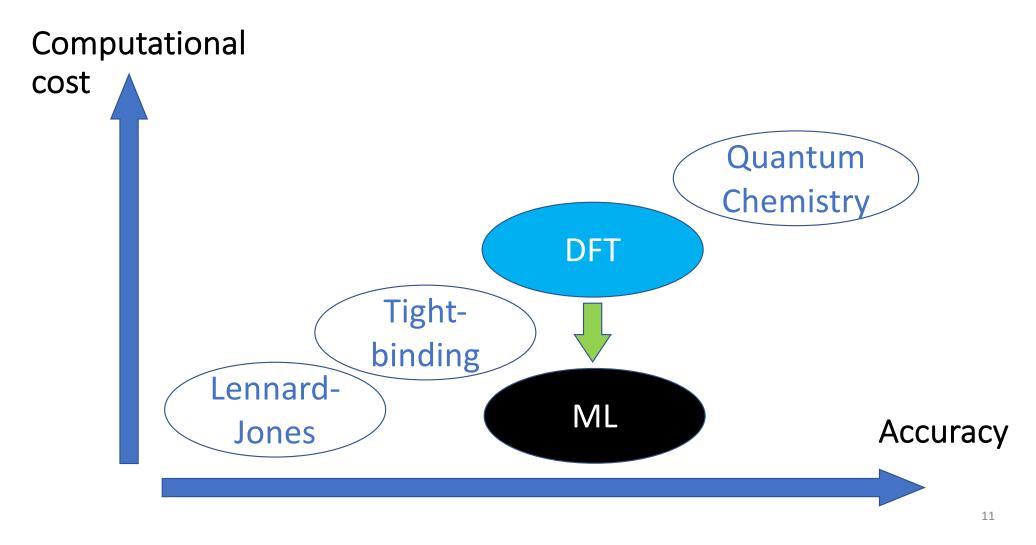
Relaxation trajectory

 $100 \times 1 \text{ hrs} = 4 \text{ days}$

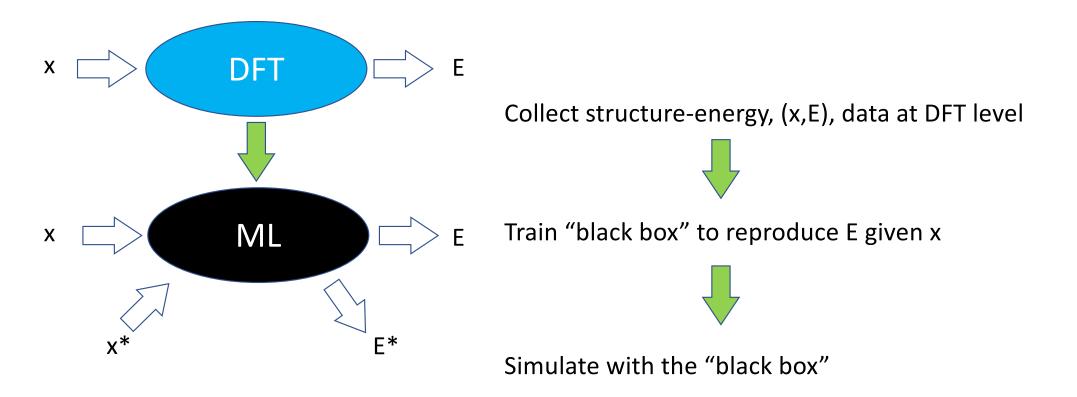
Molecular dynamics (10 ns, 0.1 fs step) 10,000 x 1 hrs = 1 year

Monte-Carlo search (100 perturbations) 100 x 100 x 1 hrs = 1 year

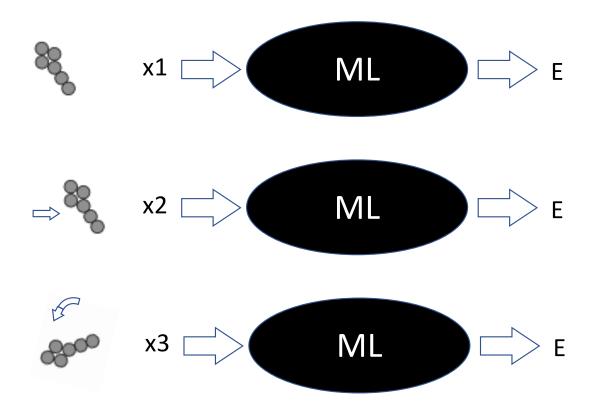
Machine learning based total energy



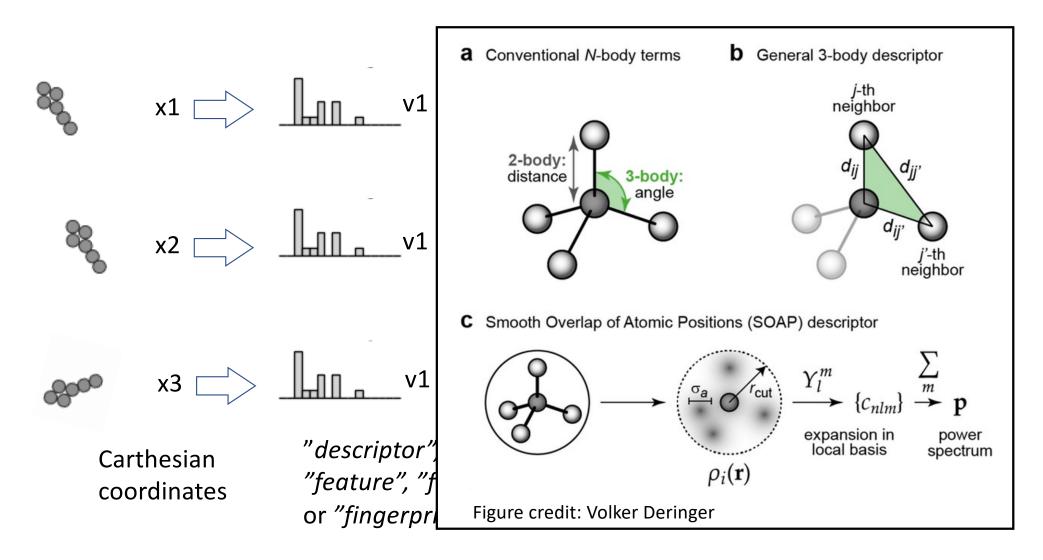
Machine learning based total energy



Expected invariant behavior of "black box"

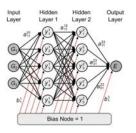


Representation



Two popular options for the "black box"

2007: Artificial Neural Networks



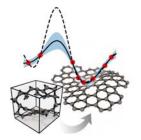
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Generalized Neural-Network Representation of High-Dimensional Potential-Energy Surfaces

Jörg Behler and Michele Parrinello Phys. Rev. Lett. **98**, 146401 – Published 2 April 2007

Citing Articles (1,783)

2010: Gaussian Process Regression



PHYSICAL REVIEW LETTERS								
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Featured in Physics

Gaussian Approximation Potentials: The Accuracy of Quantum Mechanics, without the Electrons

Albert P. Bartók, Mike C. Payne, Risi Kondor, and Gábor Csányi Phys. Rev. Lett. **104**, 136403 – Published 1 April 2010

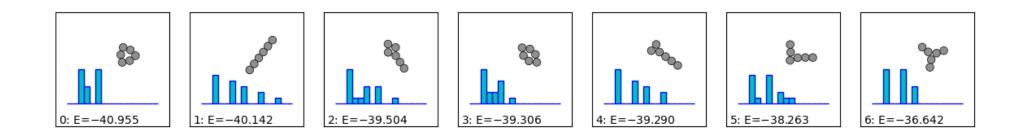
Citing Articles (1,227)

Gaussian Process Regression: Efficient with very little data

$$E_{sur}(\mathbf{x}_*) = \mathbf{k}_*^T \left(K + \sigma_n^2 I \right)^{-1} \mathbf{E}$$

$$k(\mathbf{x}_i, \mathbf{x}_j) = \theta_0 \mathrm{e}^{-||\mathbf{x}_i - \mathbf{x}_j||^2/(2\lambda)}$$

Gaussian Process Regression: Efficient with very little data



 $\mathbf{X}_{i:}$ describes the *i*'th structure

... is called: "descriptor", "feature",

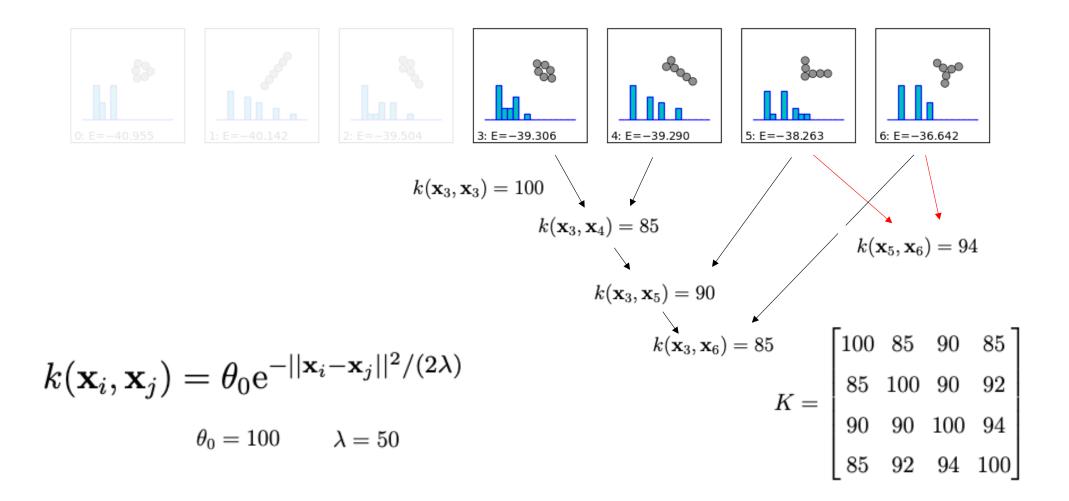
"feature vector", or *"fingerprint"*

Kernel:

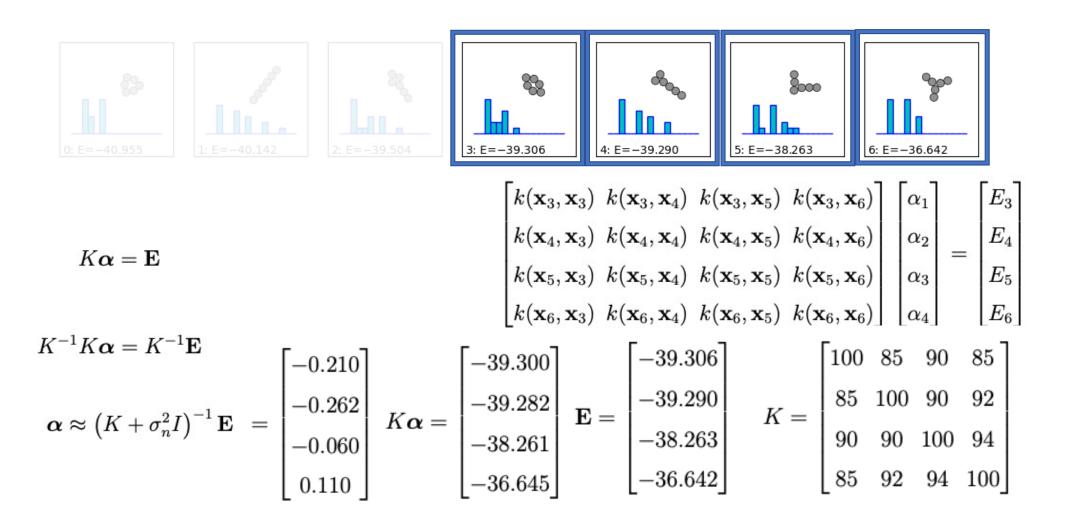
$$k(\mathbf{x}_i, \mathbf{x}_j) = \theta_0 \mathrm{e}^{-||\mathbf{x}_i - \mathbf{x}_j||^2/(2\lambda)}$$

... measures how similar two structures are

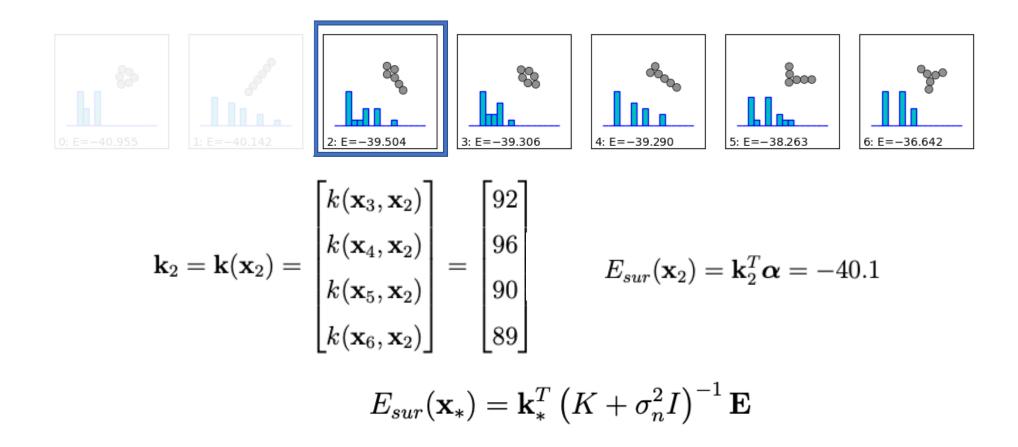
Let's say we already found structure 3, 4, 5, and 6: Construct K



Solve for model parameters and check on training data:

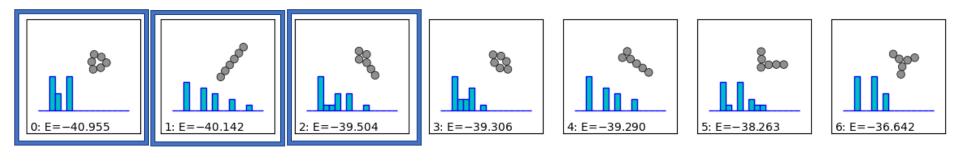


Then, for structure 2 we can construct the kernel vector:



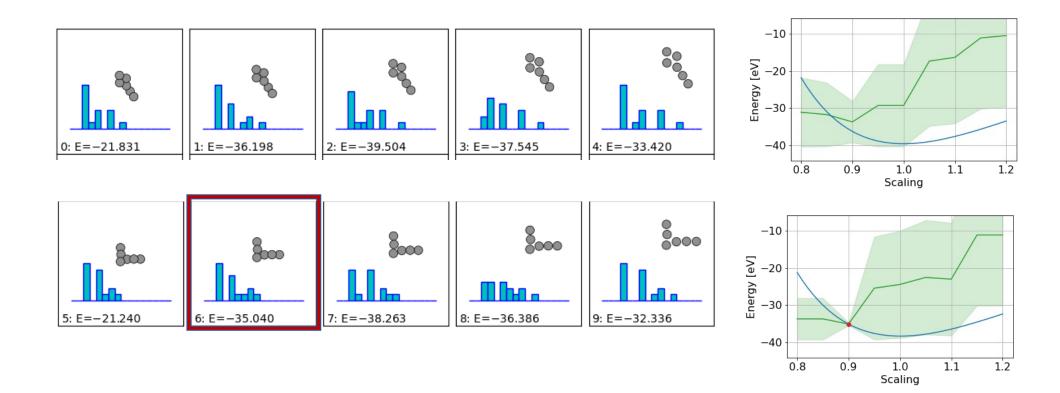
20

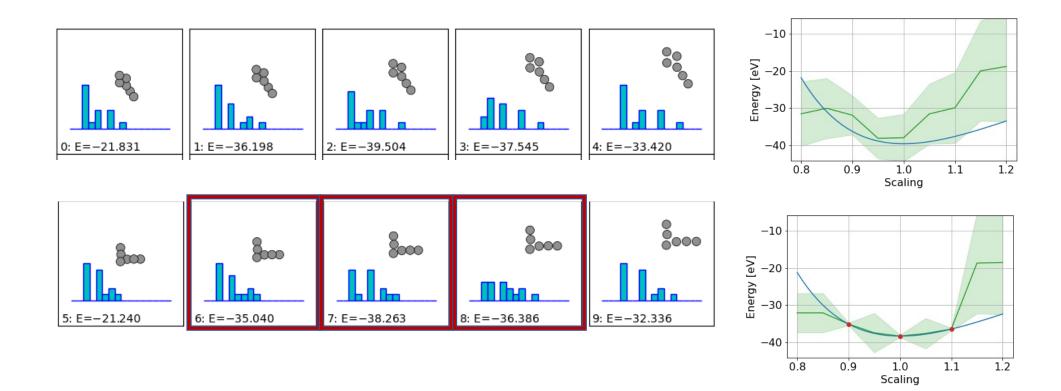
Uncertainty estimation

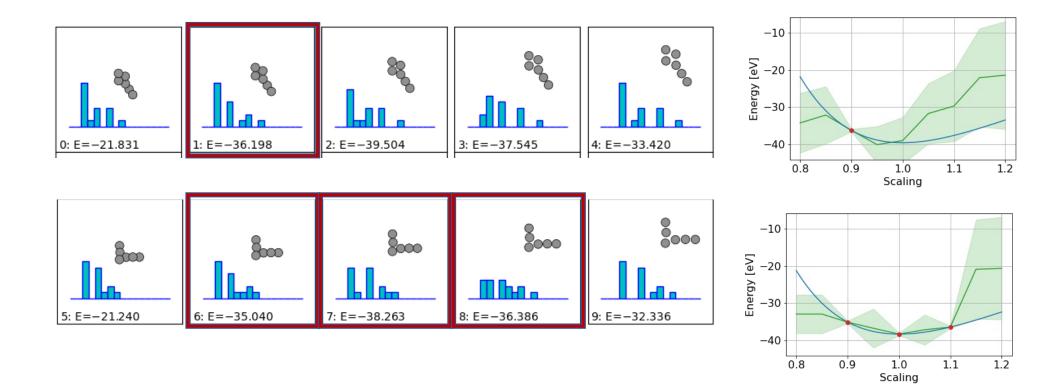


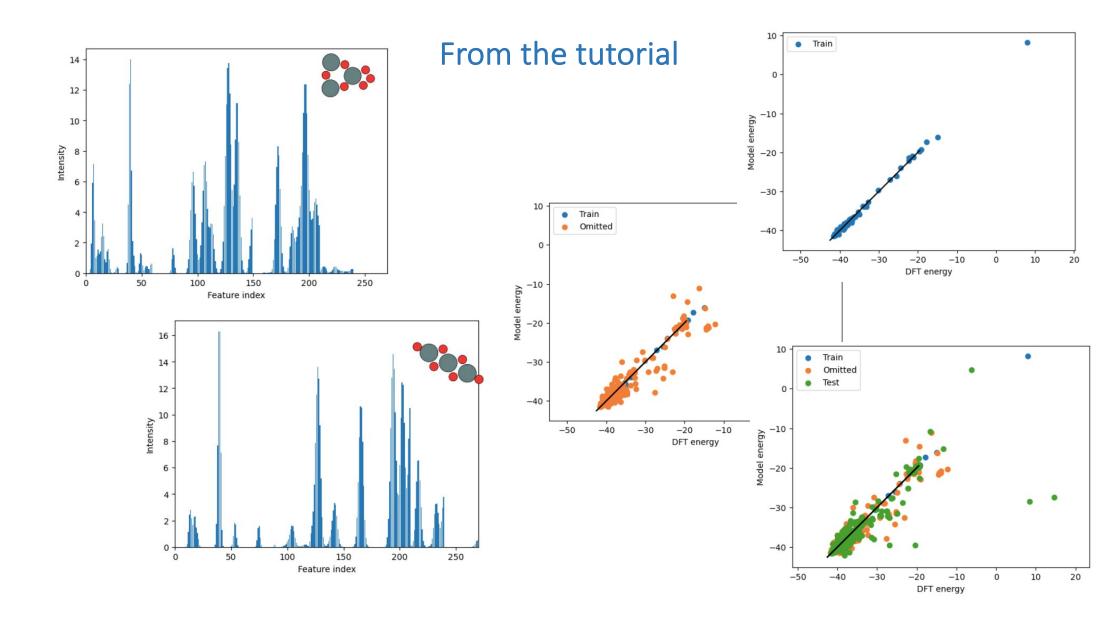
 $-35.3 \pm 3.7 \quad -38.6 \pm 1.9 \quad -40.1 \pm 1.9$

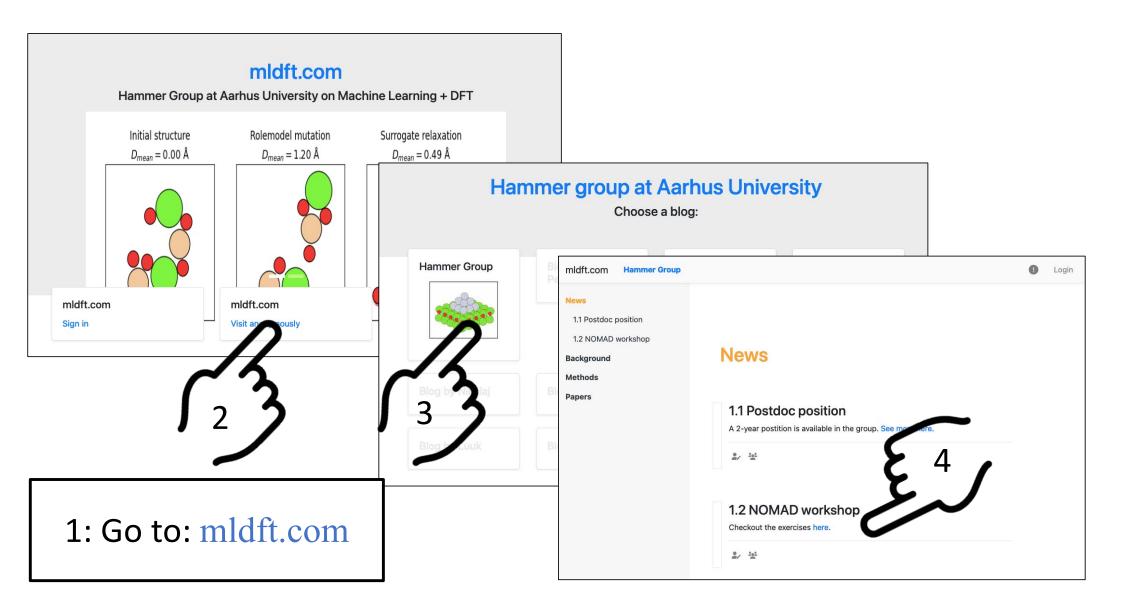
$$E_{sur}(\mathbf{x}_*) = \mathbf{k}_*^T \left(K + \sigma_n^2 I \right)^{-1} \mathbf{E}$$
$$\sigma_{sur}(\mathbf{x}_*)^2 = k(\mathbf{x}_*, \mathbf{x}_*) - \mathbf{k}_*^T (K + \sigma_n^2 I)^{-1} \mathbf{k}_*$$





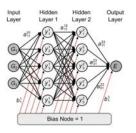






Two popular options for the "black box"

2007: Artificial Neural Networks



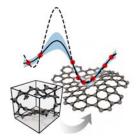
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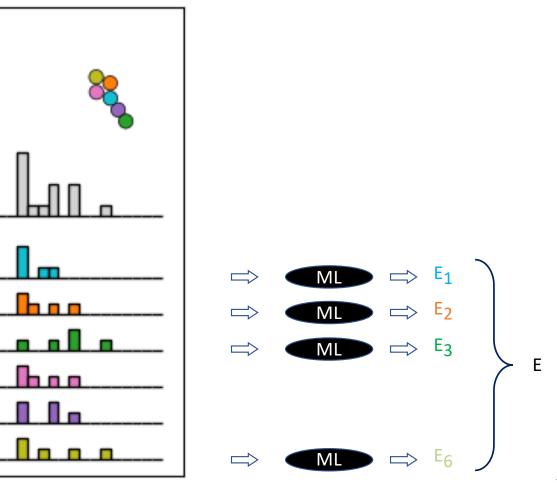
Albert P. Bartók, Mike C. Payne, Risi Kondor, and Gábor Csányi Phys. Rev. Lett. **104**, 136403 – Published 1 April 2010

Citing Articles (1,227)

In fact, both methods use: a descriptor for each atom

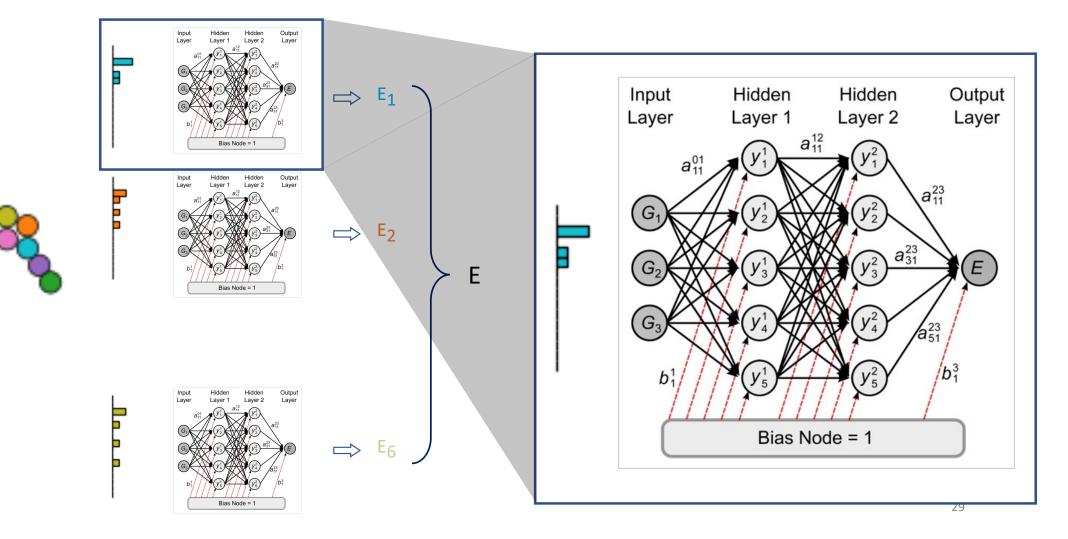
Histogram of all interatomic distances

Histograms for each atom in molecule

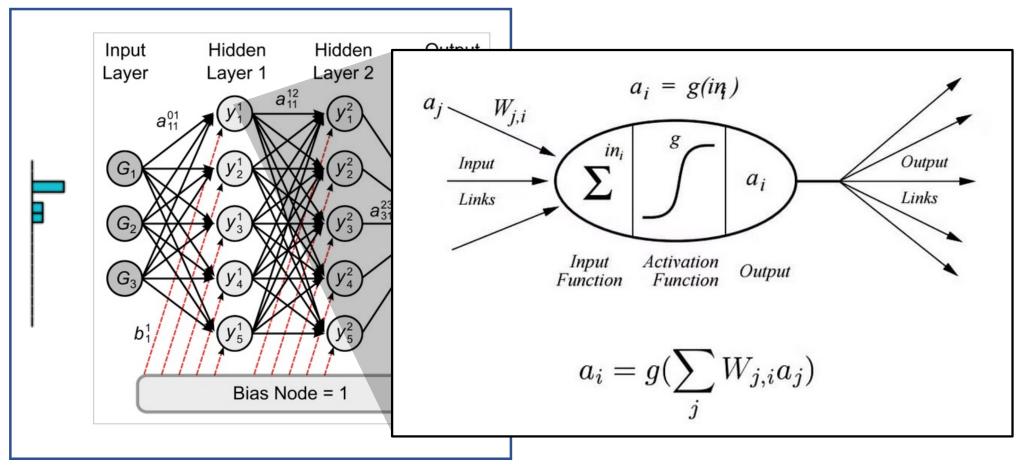


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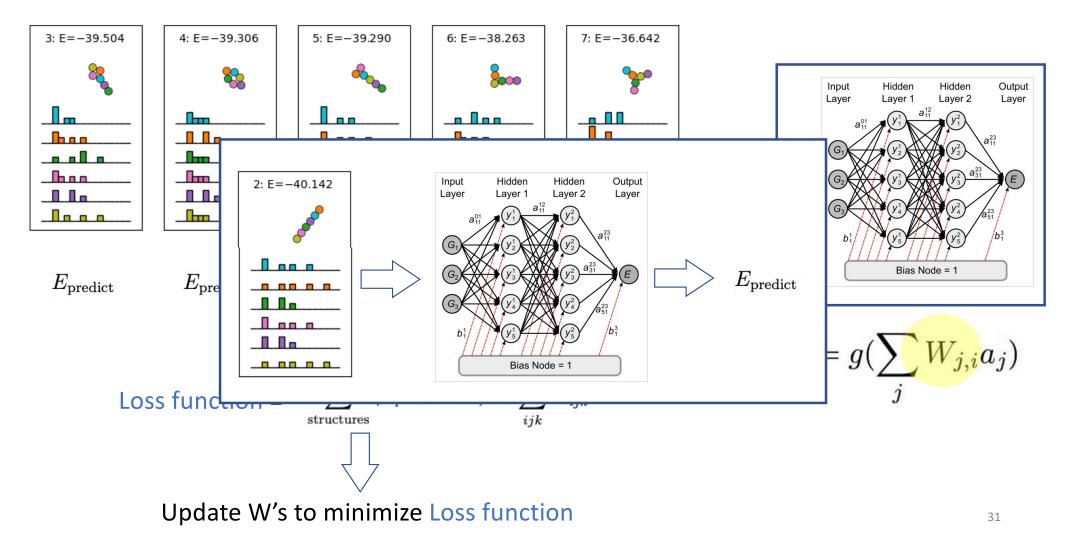
Artificial Neural Network approach

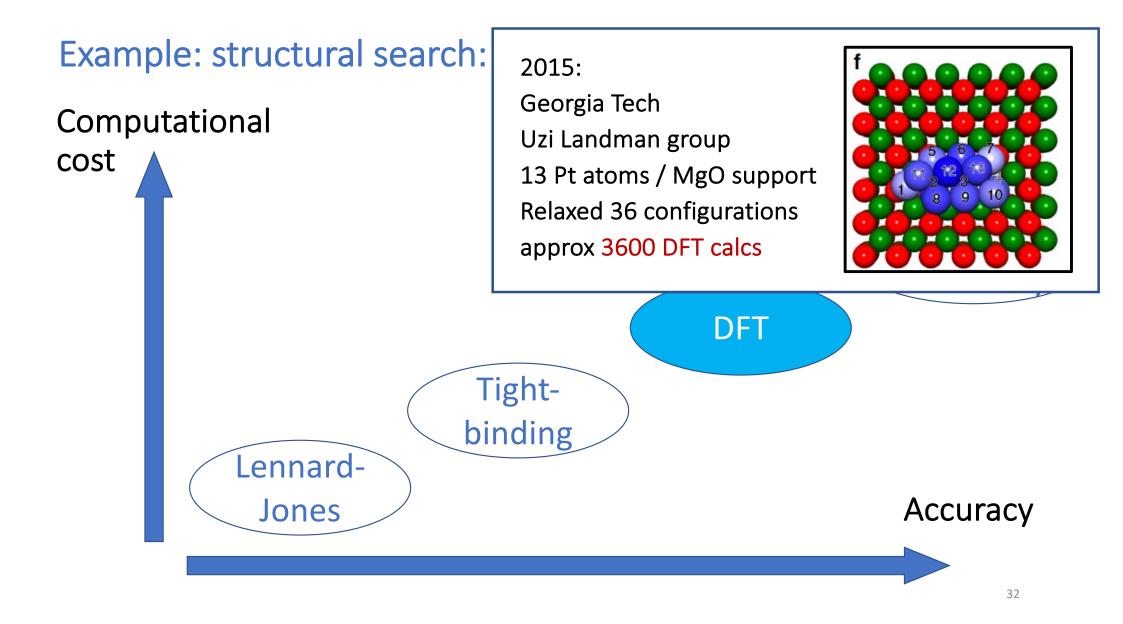


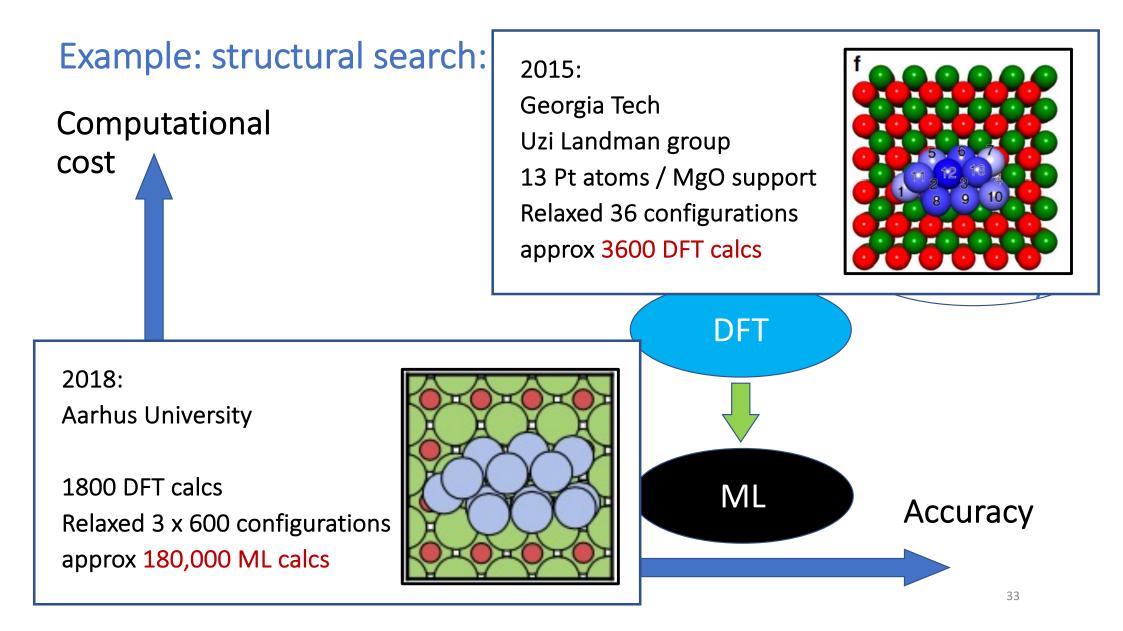
The neuron



The training



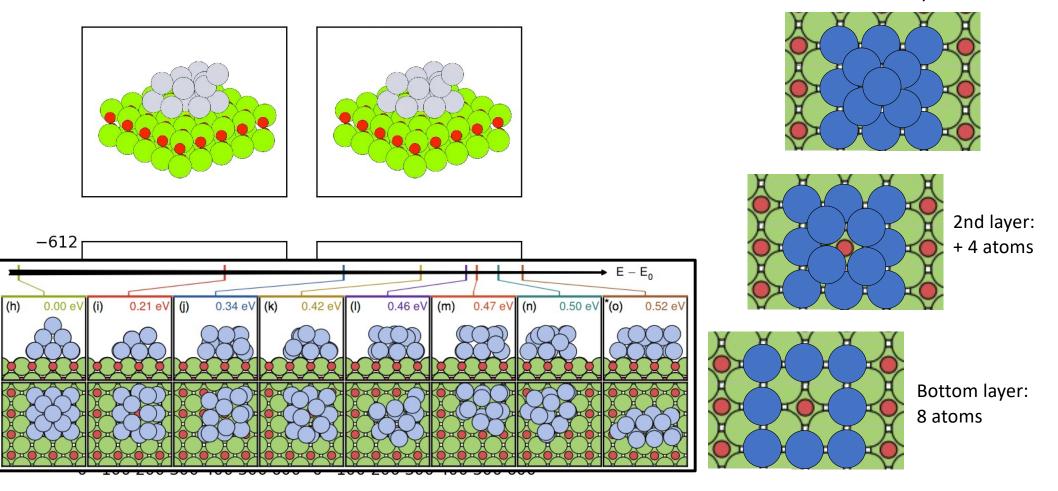


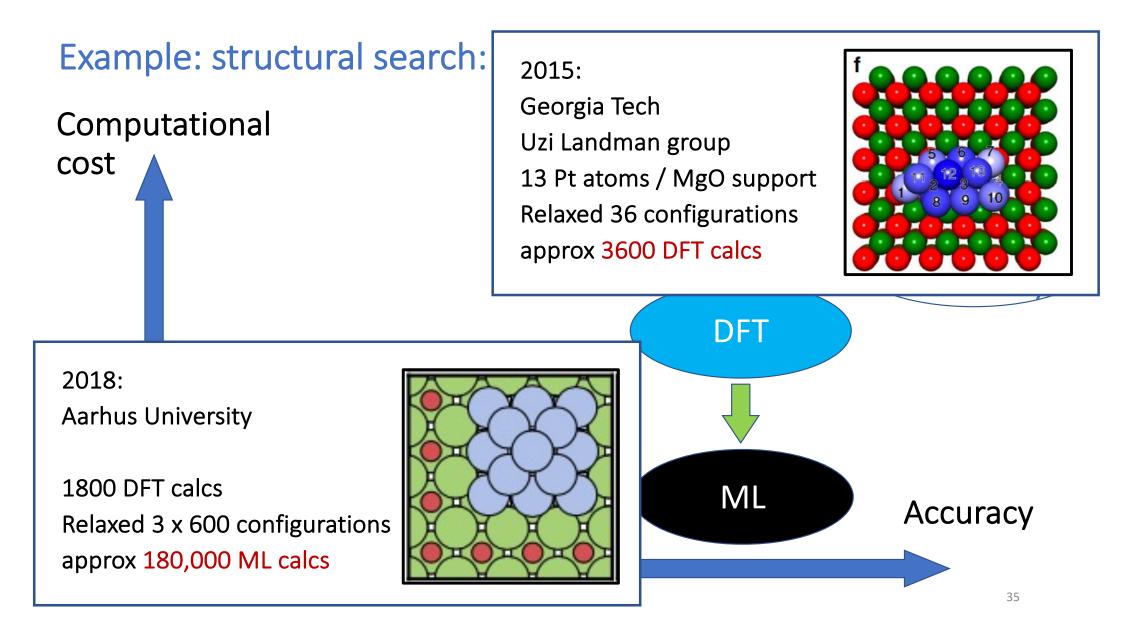


Monte-Carlo type search using ML: Pt₁₃/MgO

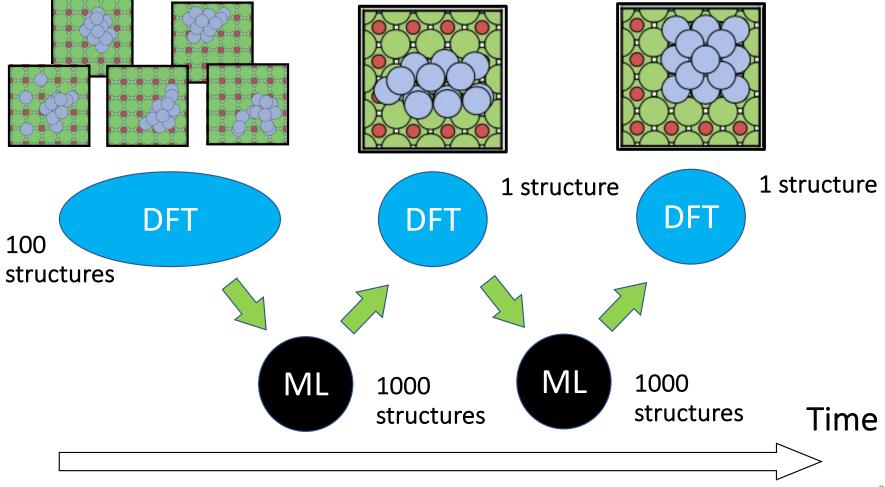
Esben Kolsbjerg, A A Peterson, BH Phys Rev B 97, 195424 (2018)

3rd layer: +1 atom



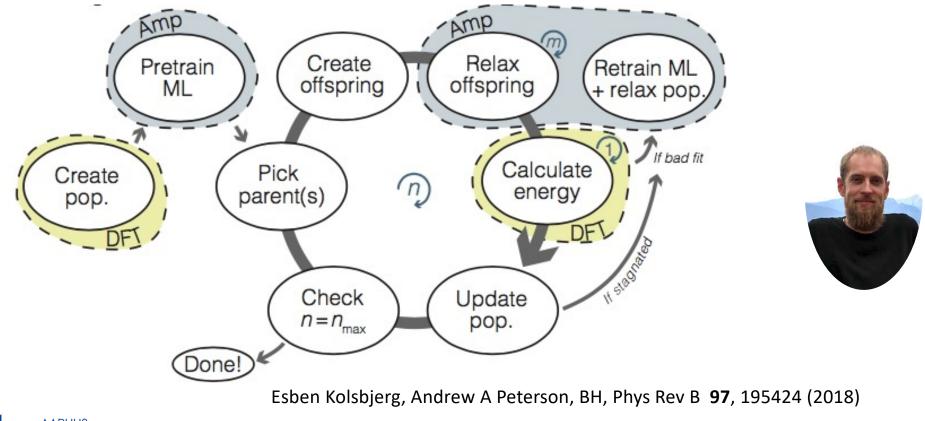


Active learning



Learning Evolutionary Algorithm: LEA

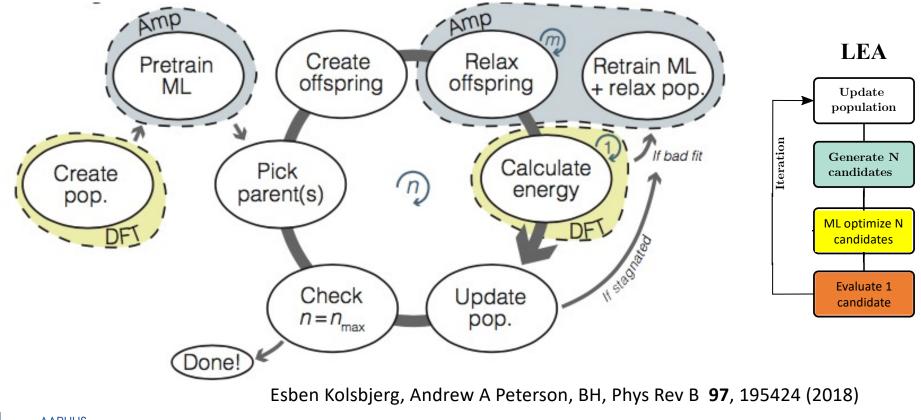
ML = Machine Learning





Learning Evolutionary Algorithm: LEA

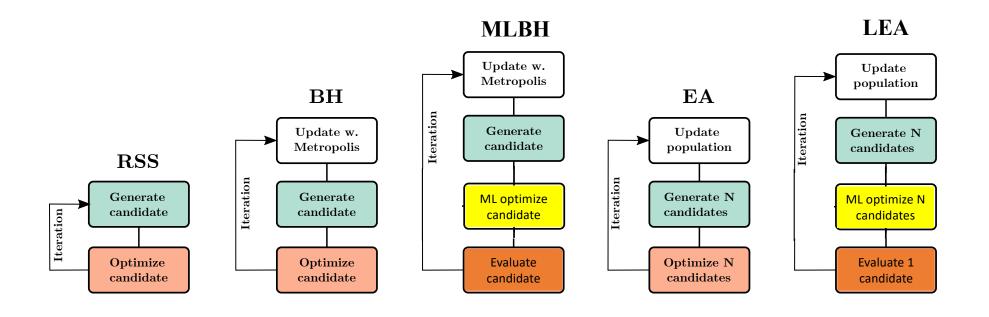
ML = Machine Learning





Common elements in Atomistic Global Optimization: AGOX

Christiansen et al, JCP 157, 054701 (2022)

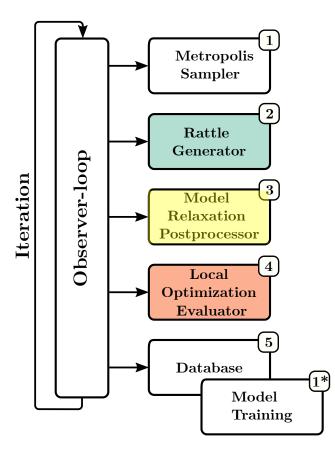


AGOX: "Atomistic Global Optimization X"

Christiansen et al, JCP 157, 054701 (2022)

https://gitlab.com/agox

ML-enhanced basin-hopping



Define a model:

model = ModelGPR.default(environment, database)

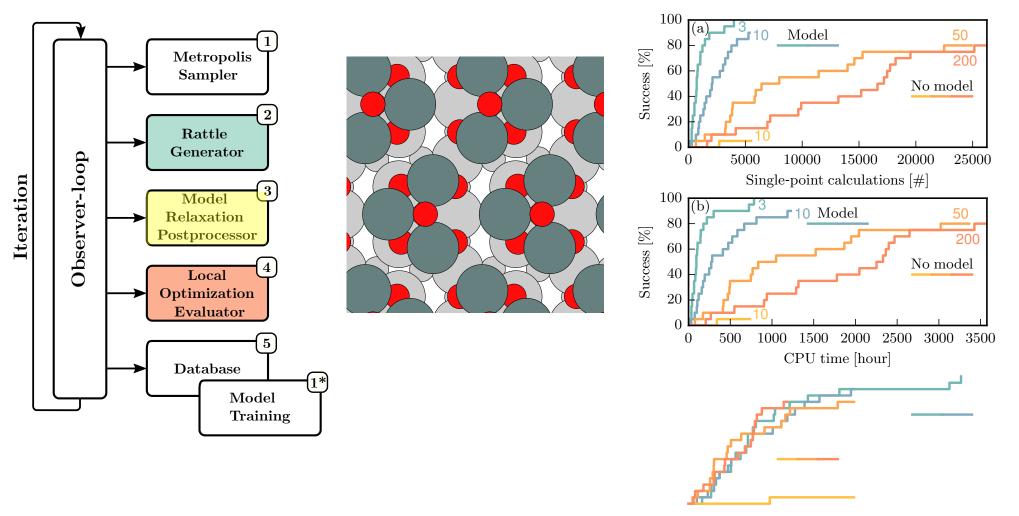
Define a module that uses this model:

```
relaxer = RelaxPostprocess(model=model,
    start_relax=10,
    optimizer_run_kwargs={'fmax':0.05, 'steps':100},
    optimizer_kwargs ={'logfile':'-'},
    optimizer=BFGS, constraints=[BC], order=2)
```

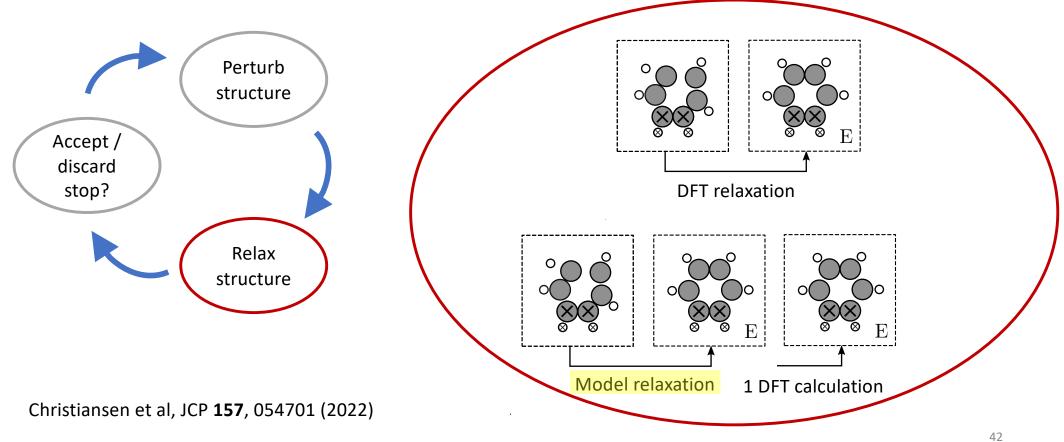
Give to AGOX

Basin hopping search with local relaxation in GPR model

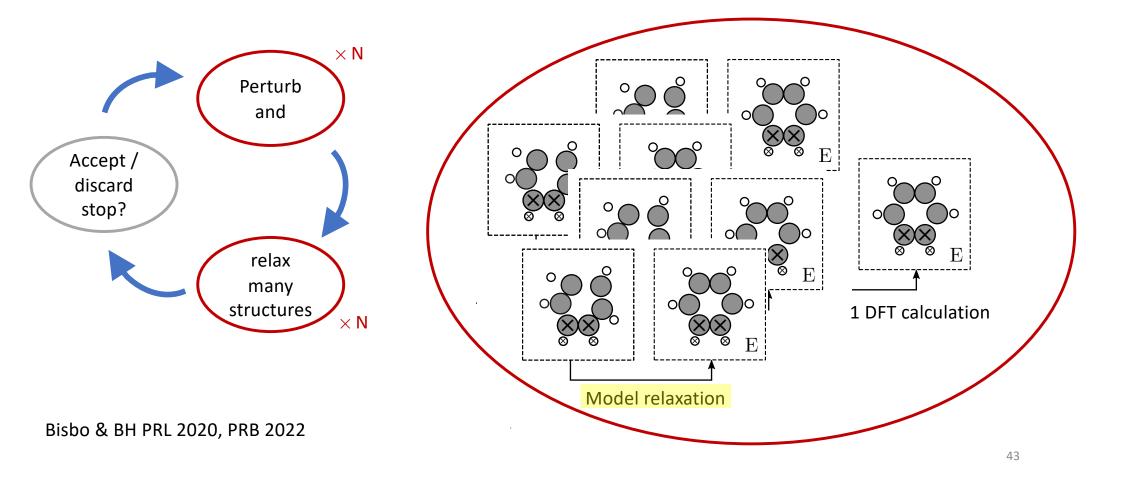
ML-enhanced basin-hopping

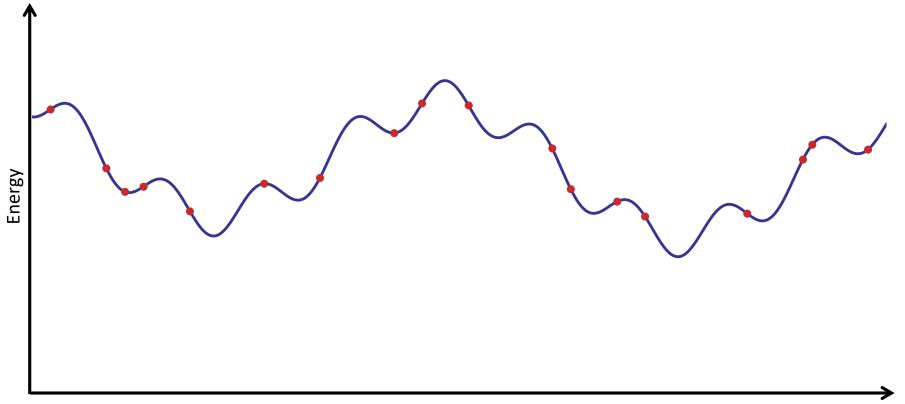


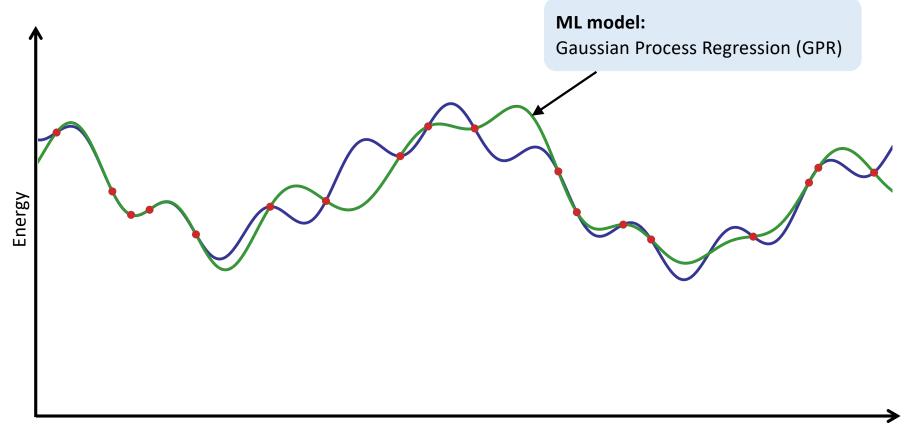
Structural search: Trick of relaxing in surrogate potential

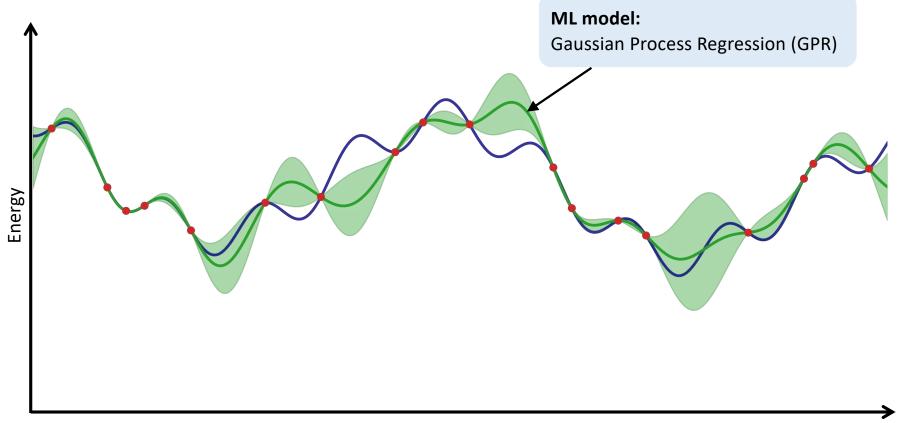


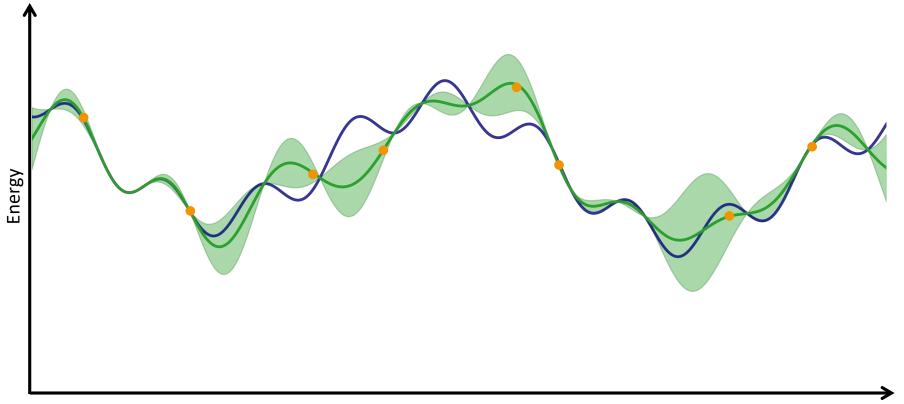
Global optimization of first-principles energy expressions (GOFEE)

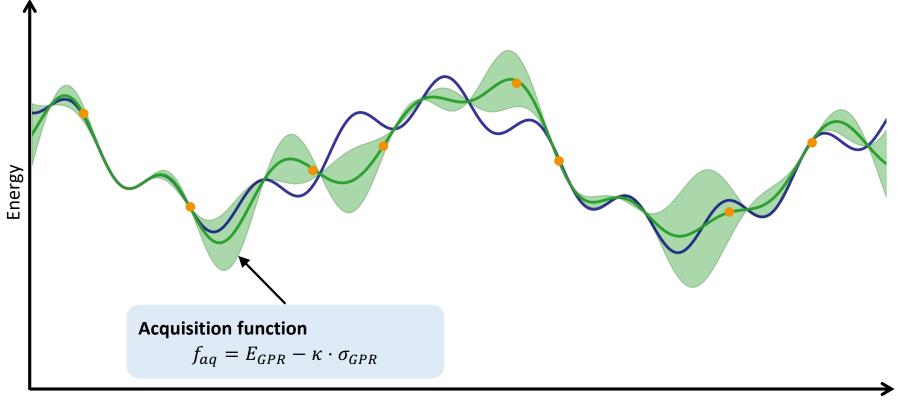


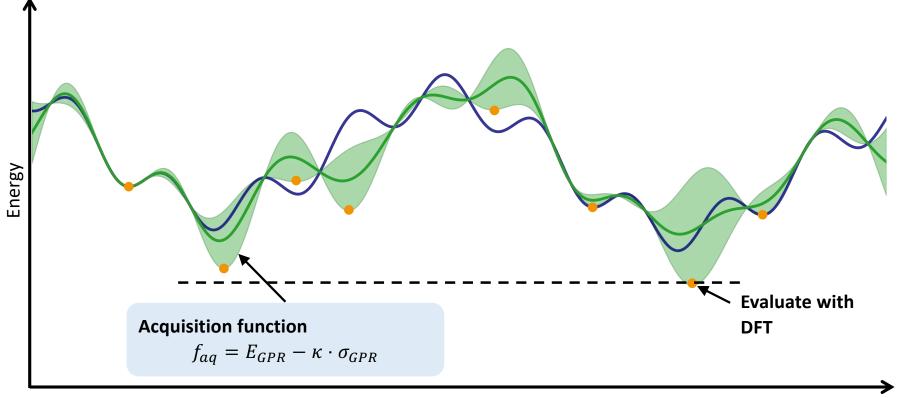




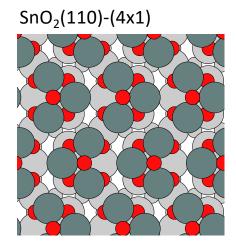


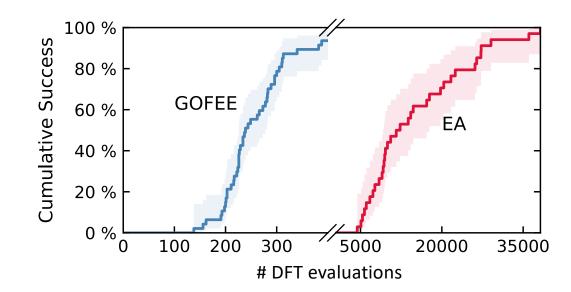






Results – the reduced SnO₂ surface reconstruction

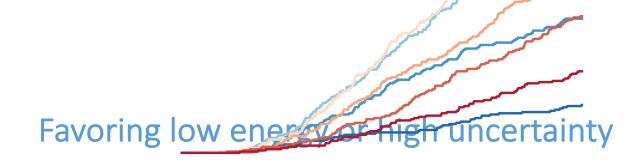




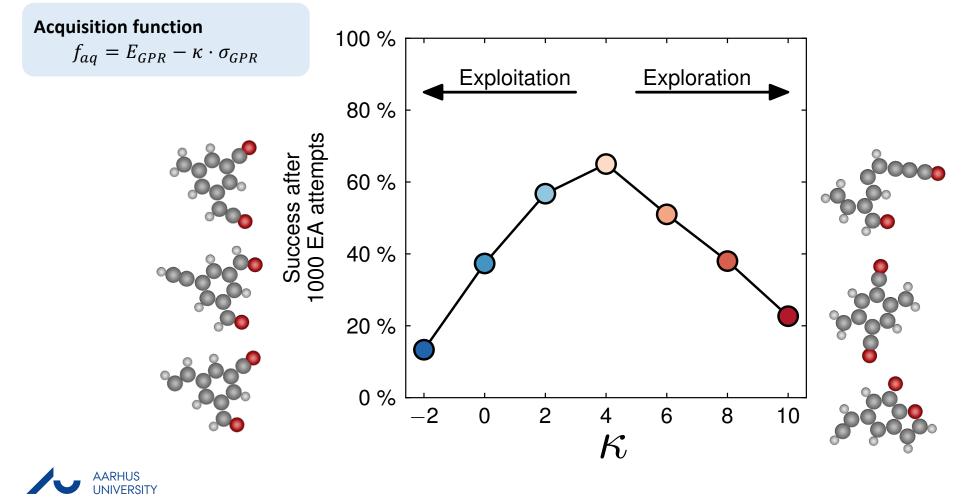
GOFEE =

- 1) Adaptive learning
- 2) Relaxation in Acquisition function: $f_{aq} = E_{GPR} \kappa \cdot \sigma_{GPR}$
- 3) Multiple candidates at model level, one at DFT level

Bisbo & BH PRL 2020, PRB 2022

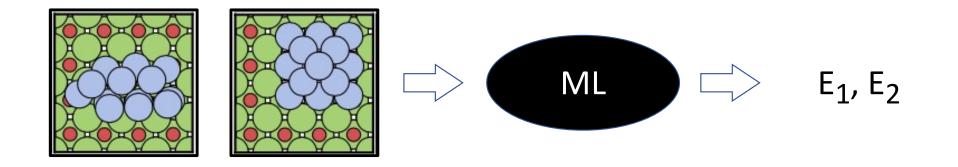


M S Jørgensen et al., J Phys Chem A, **122**, 1504 (2018)





Machine learned potential



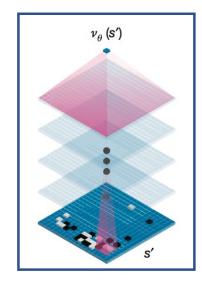
Artificial intelligence

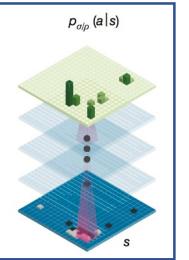


AlphaGo

Google DeepMind 2016









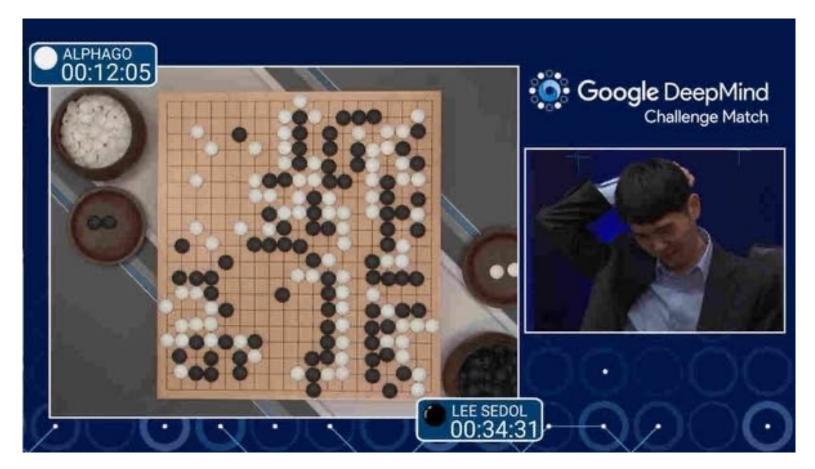
Quality assessment

Best move prediction



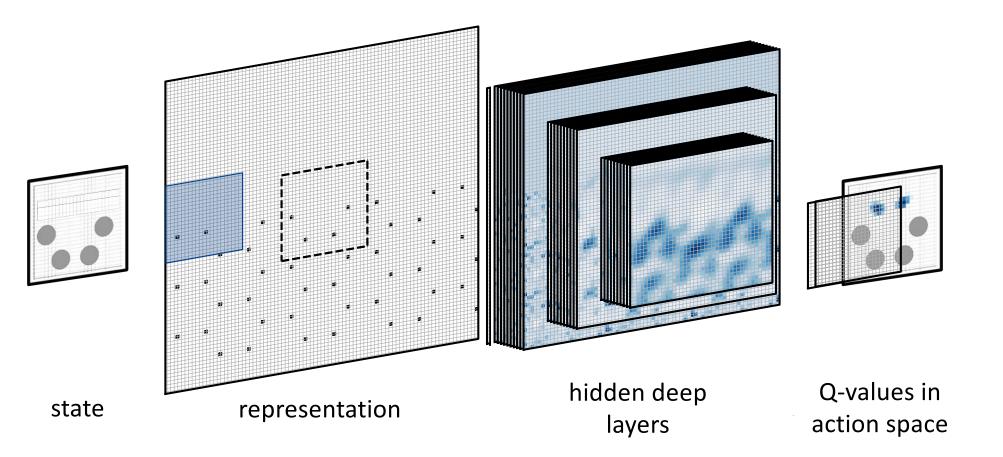
AlphaGo

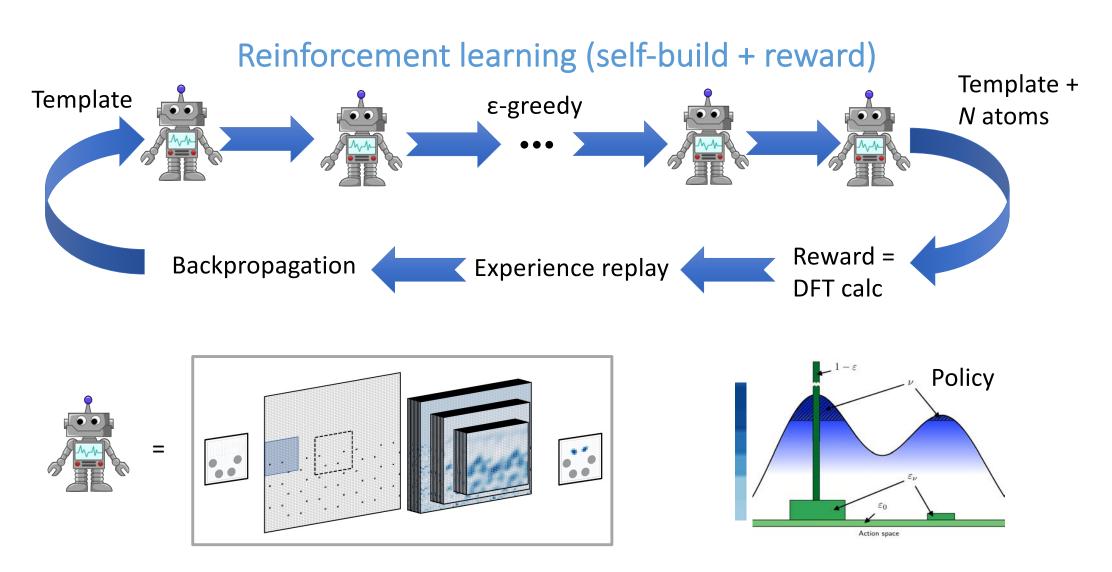
Google DeepMind 2016



Atomistic structure learning algorithm (ASLA)

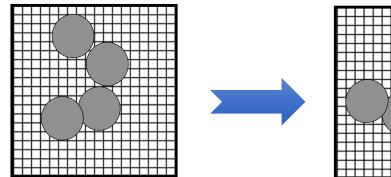
Image recognition: Convolutional Neural Network

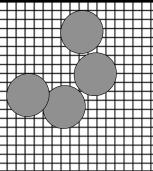


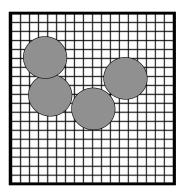


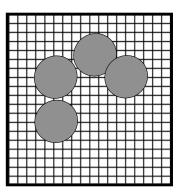


Obtain rotational invariance via data augmentation







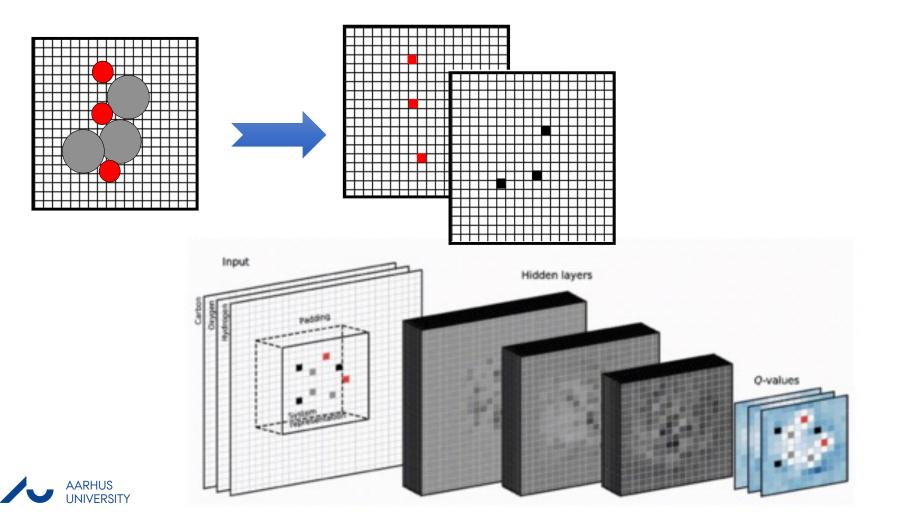


Reward evaluated based on actual DFT calculation Given same reward

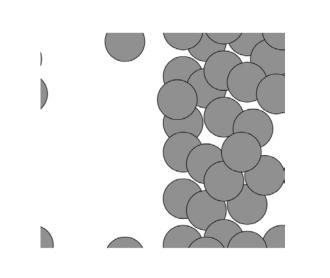
Not strictly true under periodic boundary conditions, but appears to act as regularization

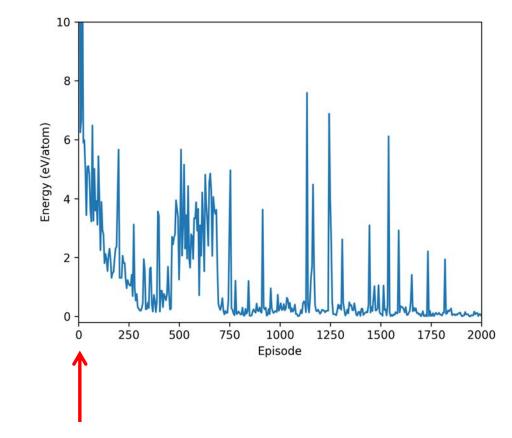


Several atom types: just add image layers – like RGB color images



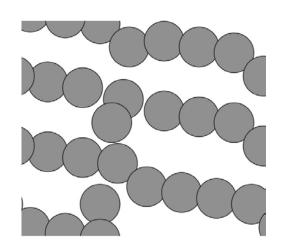
Search for graphene: untrained agent

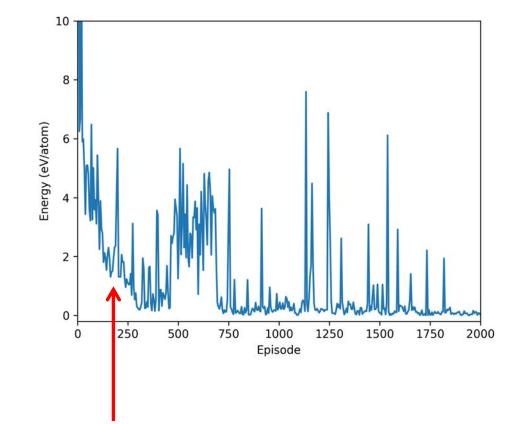






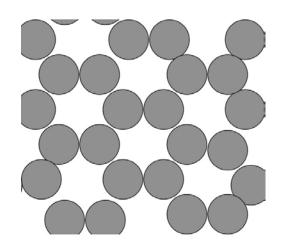
Search for graphene: agent trained for 200 episodes

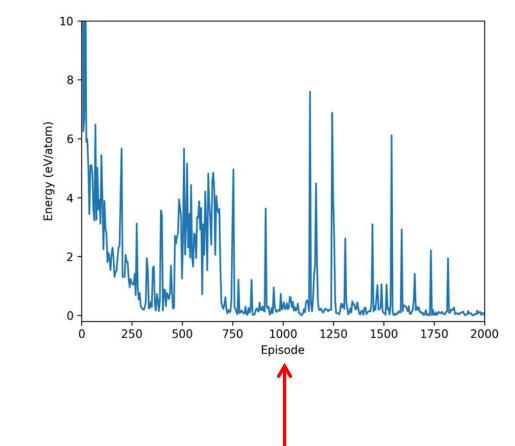






Search for graphene: agent trained for 1000 episodes







Structure of oxidized Pd(100)



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Surface Science 541 (2003) 101-112

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The Pd(100)– $(\sqrt{5} \times \sqrt{5})$ R27°-O surface oxide revisited

M. Todorova ^a, E. Lundgren ^{b,*}, V. Blum ^c, A. Mikkelsen ^b, S. Gray ^b, J. Gustafson ^b, M. Borg ^b, J. Rogal ^a, K. Reuter ^a, J.N. Andersen ^b, M. Scheffler ^a

^a Fritz-Haber-Institut der Max-Planck-Gesellschaft, Faradayweg 4-6, D-14195 Berlin, Germany ^b Department of Synchrotron Radiation Research, Institute of Physics, Lund University, Box 118, S-221 00 Lund, Sweden ^c National Renewable Energy Laboratory, Golden, CO 80401, USA

Received 26 March 2003; accepted for publication 23 June 2003

Abstract

Combining high-resolution core-level spectroscopy, scanning tunneling microscopy and density-functional theory calculations we reanalyze the Pd(100)– $(\sqrt{5} \times \sqrt{5})$ R27°-O surface oxide phase. We find that the prevalent structural model, a rumpled PdO(001) film suggested by previous low energy electron diffraction (LEED) work [Surf. Sci. 494 (2001) L799], is incompatible with all three employed methods. Instead, we suggest the two-dimensional film to consist of a strained PdO(101) layer on top of Pd(100). LEED intensity calculations show that this model is compatible with the experimental data of Saidy et al.

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Keywords: Scanning tunneling microscopy; Photoemission (total yield); Photoelectron spectroscopy; Low energy electron diffraction (LEED); Density functional calculations; Palladium; Oxygen

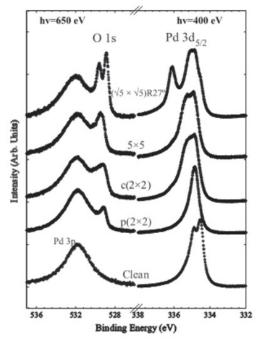
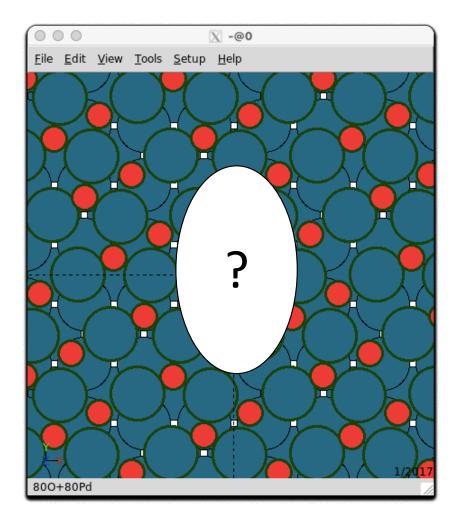


Fig. 1. Experimental HRCL spectra of the O Is and the Pd $3d_{5/2}$ levels for the sequence of ordered structures that form on Pd(100) with increasing oxygen coverage. The photon energies were 650 and 400 eV respectively.

Structure of oxidized Pd(100)

Monte-Carlo search with cheap ML energy predictions.



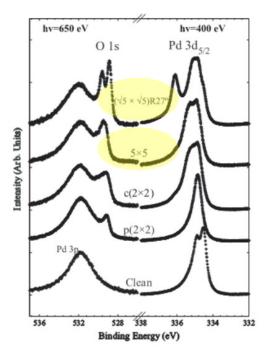
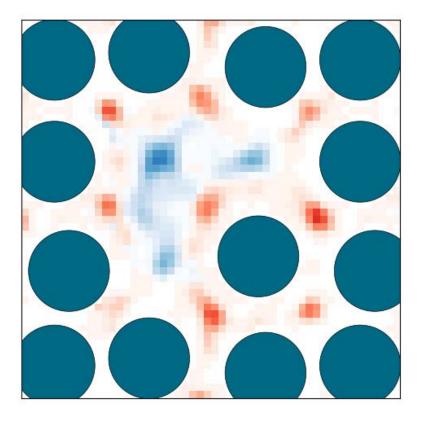
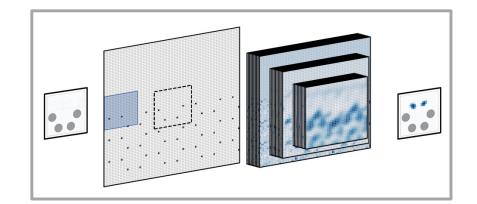


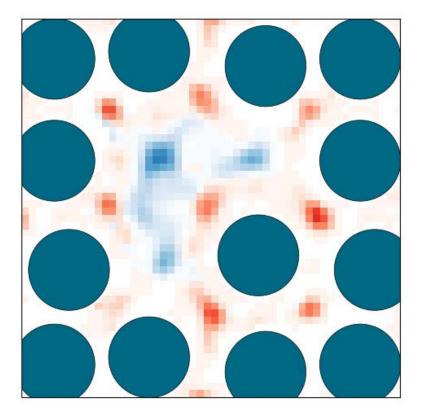
Fig. 1. Experimental HRCL spectra of the O1s and the Pd $3d_{5/2}$ levels for the sequence of ordered structures that form on Pd(100) with increasing oxygen coverage. The photon energies were 650 and 400 eV respectively.

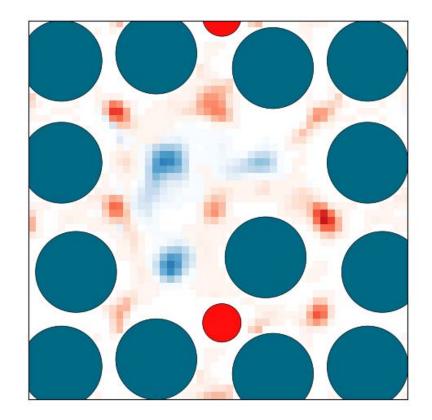
Train the ASLA agent for a (3x3) Pd_9O_6 film



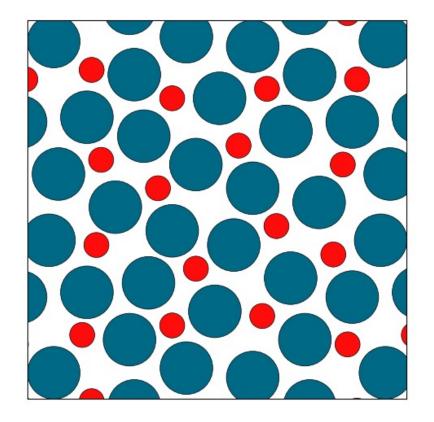


Train the ASLA agent for a (3x3) Pd_9O_6 film





Apply the ASLA agent to the (5x5) problem

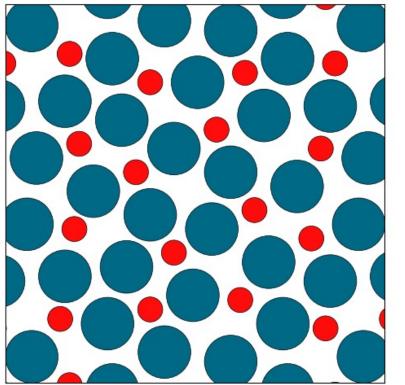


After 1000 supervised episodes on (3x3) problem and 2450 reinforcement episodes on (5x5)

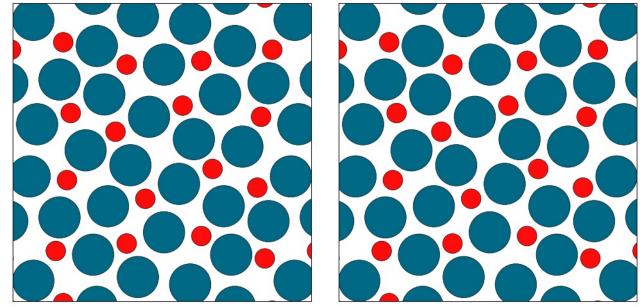
Self-taught:

- positions
- atomic types
- sequence

Apply the ASLA agent to the (5x5) problem



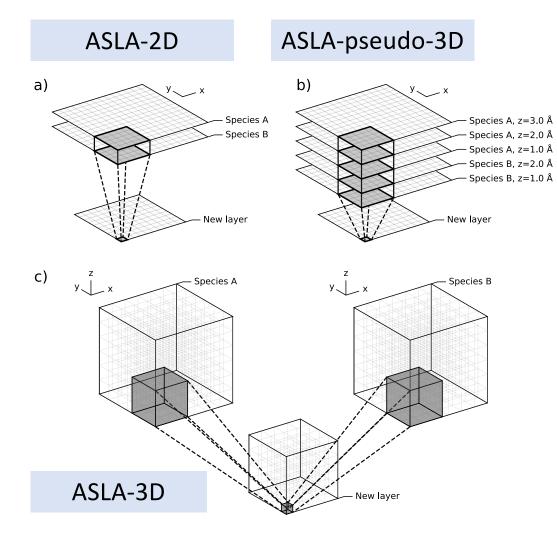
Forcing the build with two other sequences:

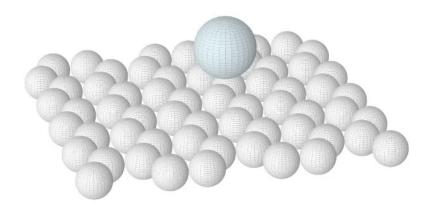


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- the agent has obtained general knowledge !

Full 3D ASLA





Pt₈ on graphene

Conclusion

Machine learning takes:

- 1) Data
- 2) Clever representation
- 3) Model

Provides:

- 1) Tremendous speedup
- 2) Intelligent solutions

Collaborators:

Lindsay Merte, Malmø Edvin Lundgren, Lund Andrew Peterson, Brown Uni Karsten W Jacobsen, DTU Jun Li, Tsinghua **Funding:** Danish National Research Council VILLUM Foundation

2019: Mortensen, Christiansen, Meldgaard, Bisbo



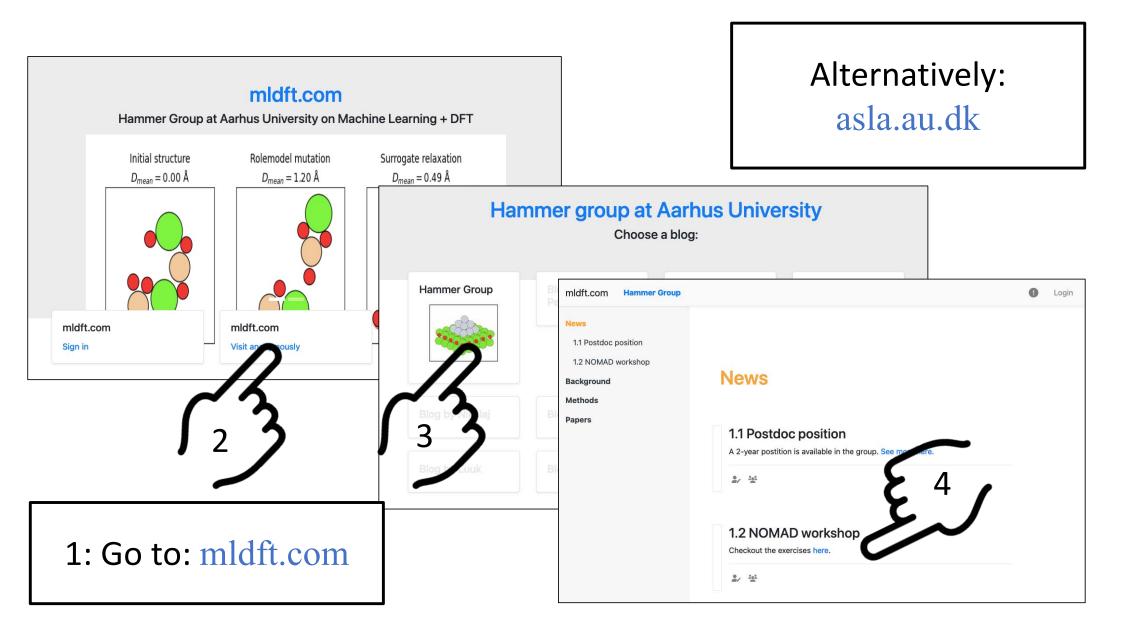




2022: Rønne, Christiansen, Brix



VILLUM FONDEN



AGOX tutorial

These are the notebooks for the AGOX tutorial / the Hammer presentation:

```
* 01_plot_training_data.ipynb
```

```
- practise basic python operations on dataset
```

```
* 02_descriptor.ipynb
```

```
- learn about descriptor
```

```
- learn about k-means
```

```
- learn about dimensionality reduction
```

```
* 03_parity_plot.ipynb
```

```
- learn about amount of data needed for model
```

```
- learn about hyperparameters
```

```
* 04_relaxation.ipynb
```

```
- see consequences for relaxations for poor model
```

```
* 05_basin_hopping.ipynb
```

```
- basin hopping with expensive target potential
```

```
- basin hopping with static cheap model
```

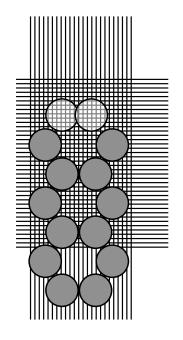
```
- basin hopping with adaptive learning, i.e. iteratively improving model
```

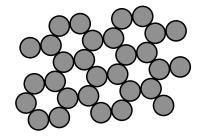
- basin hopping with lower confidence bound landscape

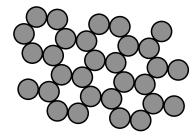
```
* 06_gofee.ipynb
```

```
- example of GOFEE method
```

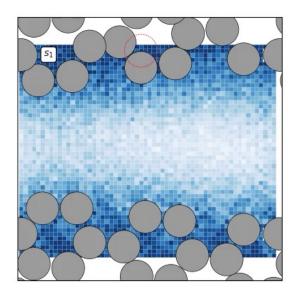
Application to grain boundary problems

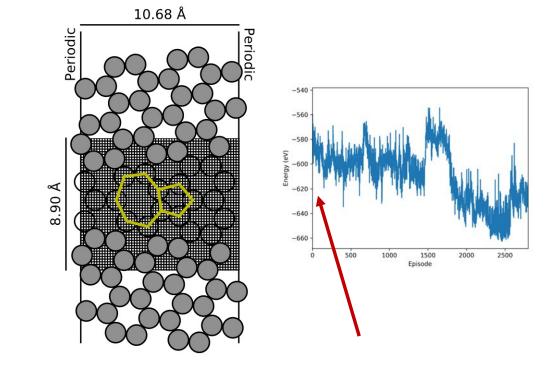




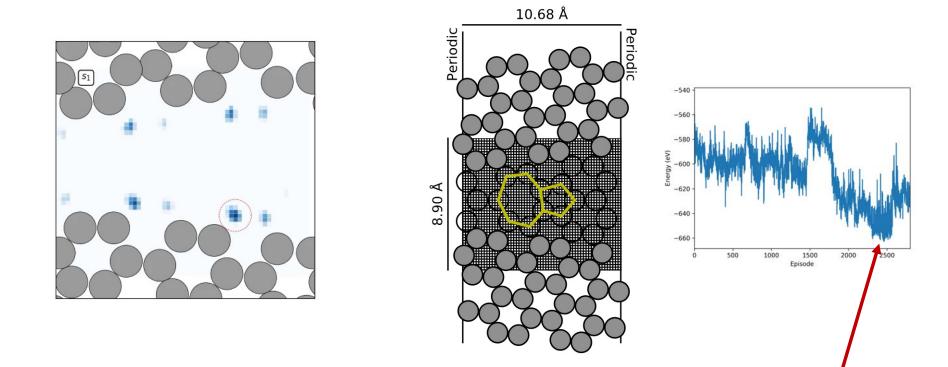


Application to grain boundary problems

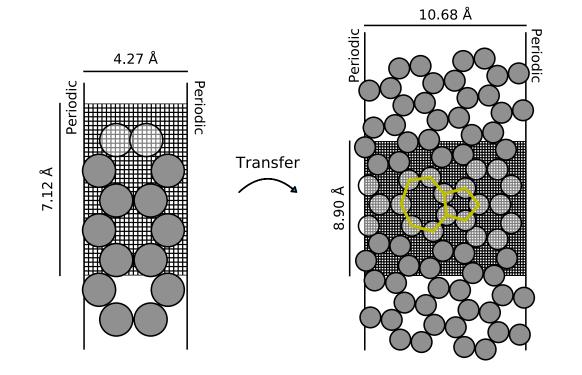




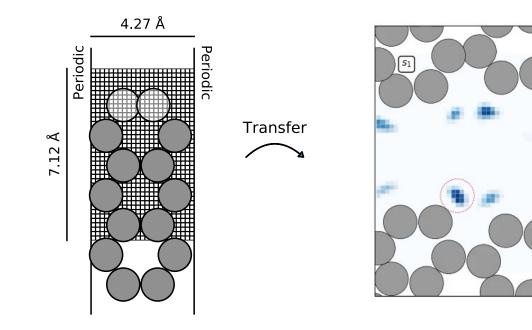
Eventually, the problem is solved



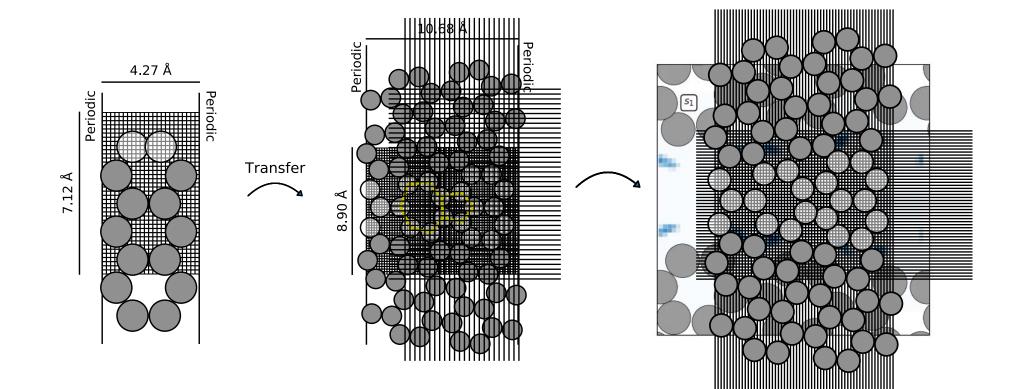
Learn elements from a simpler problem



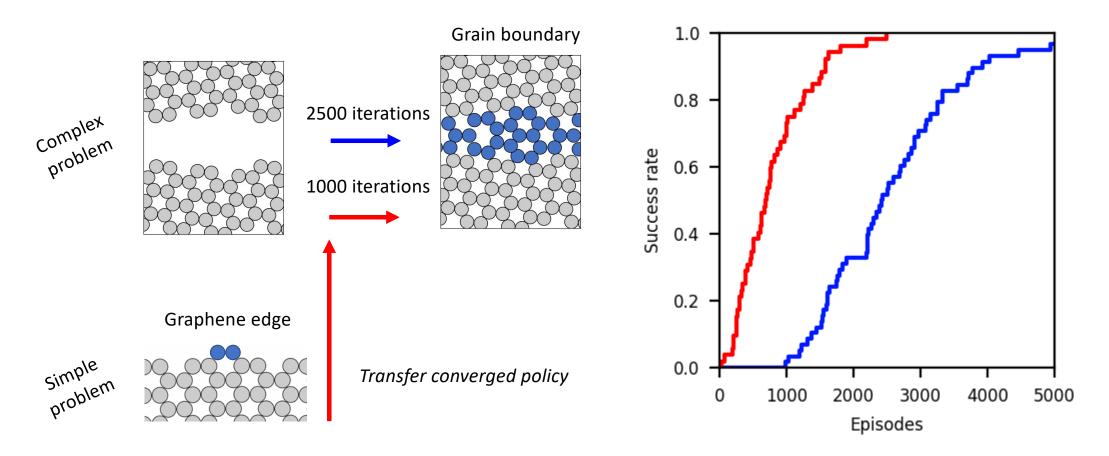
Right after transfer – something was learnt



Eventually, the problem is solved



Starting from agent trained on simpler problem





Henrik Lund Mortensen

Layer-wise relevance propagation

